

Intersite Fluctuations and Spin-Charge Separation in the Extended Hubbard Model

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We study the effects of intersite RKKY-like interactions in the mixed valence regime of a two-band extended Hubbard model. This model is known to display a metallic non-Fermi liquid state with spin-charge separation in the standard $D = \infty$ limit (where interactions lead to purely on-site fluctuations only). Using an extended $D = \infty$ approach, we find that the spin-charge separation survives the quantum fluctuations associated with arbitrary intersite density-density interactions and a finite range of intersite spin-exchange interactions. We determine the qualitative behavior of the spin, charge, and single-particle correlation functions. The implications of these results for the solution in finite spatial dimensions are discussed.

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Whether or not spin-charge separation exists in two or three dimensions is an important outstanding question in the area of strongly correlated electron systems. Spin-charge separation occurs in the Luttinger liquid, which describes interacting fermion systems in 1D [1]. The Luttinger-liquid picture breaks down in $D > 1$ when interactions are treated perturbatively [2]. What happens when interactions are treated non-perturbatively remains an open question [3]. Recently, in the opposite $D = \infty$ limit a non-Fermi liquid state with spin-charge separation has been identified in the mixed-valence regime of the two-band extended Hubbard model [4–7]. Named the intermediate phase, this state has quasiparticle-like spin excitations and incoherent single particle and charge excitations.

A natural and important question at this point is whether the spin-charge-separation phenomenon persists in finite dimensions. Here, we address this issue using the extended $D = \infty$ approach [8,9]. The key observation motivating this approach is that, the most important ingredients that exist in finite dimensions but are missing in the standard $D = \infty$ limit are the quantum fluctuations associated with intersite interactions. By introducing an appropriate scaling of the intersite two-particle interactions in terms of $1/D$, the extended $D = \infty$ approach treats the intersite fluctuations on an equal footing with on-site interactions. We note that several alternative approaches [10,11] have also been proposed to go beyond the standard $D = \infty$ dynamical mean field theory; they appear to be less practical at this stage.

The two-band extended Hubbard model we study is defined by the following Hamiltonian,

$$\begin{aligned}
H &= \sum_i h_i + \sum_{\langle ij \rangle} h_{ij} \\
h_i &= \epsilon_d^0 n_{d_i} + U n_{d_i\uparrow} n_{d_i\downarrow} + \sum_{\sigma} t (d_{i\sigma}^{\dagger} c_{i\sigma} + H.c.) + V n_{d_i} n_{c_i} + J_K \vec{S}_{d_i} \cdot \vec{s}_{c_i} \\
h_{ij} &= t_{ij} \sum_{\sigma} c_{i\sigma}^{\dagger} c_{j\sigma} + v_{ij} : n_{d_i} : : n_{d_j} : + J_{ij} \vec{S}_{d_i} \cdot \vec{S}_{d_j}
\end{aligned} \tag{1}$$

There are two kinds of electrons, the strongly correlated d -electrons and the band c -electrons. Both have spin $1/2$, with n_{d_i} , n_{c_i} , \vec{S}_{d_i} and \vec{s}_{c_i} representing the corresponding density and spin operators at the site i . The Hamiltonian is written as the sum of two parts. h_i describes purely local couplings at the site i . ϵ_d^0 and U are the energy level

and the Hubbard interaction respectively. For the purpose of studying the mixed-valence regime it suffices to take U to be infinite. t is the hybridization. Also included are two more local interaction terms allowed by symmetry (which are neglected in the standard Anderson lattice model): V is a charge-screening interaction and J_K is a spin-exchange interaction. h_{ij} describes three kinds of intersite couplings, including the single-particle hopping (t_{ij}), the two-particle RKKY-like density-density interaction (v_{ij}) and spin-exchange interaction (J_{ij}).

The extended large D limit is taken by scaling the two-particle nearest-neighbor interaction terms $v_{<ij>} = v_0/\sqrt{2D}$ and $J_{<ij>} = J_0/\sqrt{2D}$, along with the usual scaling for the single-particle hopping $t_{<ij>} = t_0/\sqrt{2D}$, while keeping v_0 , J_0 , and t_0 fixed. This limit is well defined when the static (Hartree) terms are subtracted by introducing the normal ordering: $:n:\equiv n - \langle n \rangle$. We will limit our discussion to the non-ordered states. In the $D = \infty$ limit, all the correlation functions of the lattice model can be calculated from an impurity coupled to self-consistent media. The effective impurity action can be derived using the “cavity method” [7,8]. One first selects an arbitrary site 0 on the lattice and expands the partition function in terms of all non-zero $h_{i0} + h_{0i}$, each of which is of order $1/\sqrt{D}$. One then integrates out all the degrees of freedom at sites other than site 0, and takes the $D = \infty$ limit. The resulting effective impurity action is as follows,

$$S_{imp}^{eff} = S_0 - \int_0^\beta d\tau d\tau' [\sum_\sigma c_{0,\sigma}^\dagger(\tau) G_0^{-1}(\tau - \tau') c_{0,\sigma}(\tau') + :n_{d_0}:(\tau) \chi_{c,0}^{-1}(\tau - \tau') :n_{d_0}:(\tau') + \vec{S}_{d_0}(\tau) \chi_{s,0}^{-1}(\tau - \tau') \cdot \vec{S}_{d_0}(\tau')] \quad (2)$$

where S_0 is the action associated with h_0 , and G_0^{-1} , $\chi_{c,0}^{-1}$, and $\chi_{s,0}^{-1}$ are determined self-consistently by the single particle Green function, the two-particle density-density and spin-spin correlation functions, respectively. The impurity action (2) can be equivalently written in terms of the following impurity Hamiltonian,

$$H_{imp}^{eff} = H_{kin} + E_d^0 n_{d_0} + U n_{d_0\uparrow} n_{d_0\downarrow} + V n_{d_0} n_{c_0} + J_K \vec{S}_{d_0} \cdot \vec{s}_{c_0} + t \sum_{k\sigma} (d_{0\sigma}^\dagger c_{k\sigma} + H.c.) + F \sum_q :n_{d_0}: (\rho_q + \rho_{-q}^\dagger) + g \sum_q \vec{S}_{d_0} \cdot (\vec{\phi}_q + \vec{\phi}_{-q}^\dagger) \quad (3)$$

Eq. (3) describes an impurity coupled to three non-interacting bands. The impurity has an energy level $E_d^0 = \epsilon_d^0 - \mu$, where μ is the chemical potential, and a Hubbard interaction U . The three bands are, respectively, fermionic, scalar-bosonic, and vector-bosonic. We stress that the different components of the vector bosons commute with each other [12]: $[\phi_q^\mu, \phi_{q'}^{\nu\dagger}] = \delta_{\mu\nu}\delta_{qq'}$ where $\mu, \nu = x, y, z$. The dispersions of these bands are specified by $H_{kin} = \sum_{k\sigma} E_k c_{k\sigma}^\dagger c_{k\sigma} + \sum_q W_q \rho_q^\dagger \rho_q + \sum_q w_q \vec{\phi}_q^\dagger \cdot \vec{\phi}_q$. We have chosen E_k , W_q , and w_q such that the couplings of the impurity to the three bands are independent of momentum. In the mixed-valence regime, the self-consistent density of states of the fermionic bath at the Fermi energy, ρ_0 , turns out to be always finite. This is a crucial feature that allows for the following asymptotic analysis. Without loss of generality, we will in addition consider only the case of the Bethe lattice with infinite connectivity. Here, the self-consistency equations for the scalar- and vector- bosonic baths have the following forms:

$$\begin{aligned} F^2 \sum_q \frac{W_q}{(i\nu_n)^2 - W_q^2} &= v_0^2 \chi_c(i\nu_n) \\ g^2 \sum_q \frac{w_q}{(i\nu_n)^2 - w_q^2} &= J_0^2 \chi_s(i\nu_n) \end{aligned} \quad (4)$$

where χ_c and χ_s are the local dynamical charge and spin susceptibilities, respectively.

When intersite couplings (J_0 and v_0) are absent, F and g vanish in H_{imp}^{eff} . The corresponding self-consistent problem has been solved asymptotically exactly [4]. When J_K is antiferromagnetic, there is a phase transition between the usual strong-coupling phase, a Fermi liquid, and the intermediate phase, a non-Fermi liquid with spin-charge separation. In the presence of intersite couplings, both F and g become non-zero. The impurity d orbital is then coupled to the additional scalar-bosonic and vector-bosonic baths. Two effects of these additional baths have to be addressed. First, do they affect the coupling between the spin and charge degrees of freedom such that the spin-charge-separation phenomenon is destroyed? Second, if spin-charge separation persists, what are the forms of the spin, charge, and single-particle correlation functions?

To address these questions, we need first to know the solution to the impurity Hamiltonian, H_{imp}^{eff} , for generic forms of W_q and w_q (E_k is such that ρ_0 is always finite). Anticipating

the scale-invariant forms for the charge and spin susceptibilities, we consider the case when W_q and w_q are such that,

$$\begin{aligned}\sum_q e^{-W_q\tau} &= \frac{K_c}{\tau^{\alpha_c}} \\ \sum_q e^{-w_q\tau} &= \frac{K_s}{\tau^{\alpha_s}}\end{aligned}\tag{5}$$

Here α_c , α_s , K_c , and K_s are certain fixed parameters. (The self-consistency, Eq. (4), is not imposed at this point).

When $g = 0$ [13], the effect of the F -coupling can be determined using a renormalization group (RG) analysis based on a Coulomb-gas representation for the partition function $Z_{imp} = \text{Tr}(e^{-\beta H_{imp}^{eff}})$. In a standard fashion, we expand Z_{imp} in terms of the t - and J_K^\perp -couplings (where J_K^\perp is the transverse component of the J_K -coupling) leading to an infinite series. Each term in the series is associated with a particular history of the impurity configurations, $|0\rangle_d$ and $|\sigma\rangle_d = d_\sigma^\dagger|0\rangle_d$, along the imaginary time axis $[0, \beta)$. The procedure parallels that of Ref. [4]. The resulting RG equations [13] are as follows,

$$\begin{aligned}dy_t/d\ln\xi &= (1 - \epsilon_t - M_t)y_t + y_j y_t \\ dy_j/d\ln\xi &= (1 - \epsilon_j)y_j + y_t^2 \\ dM_t/d\ln\xi &= (2 - \alpha_c - 6y_t^2)M_t\end{aligned}\tag{6}$$

Here ξ is the running inverse energy scale, $y_t = t\xi$, and $y_j = \frac{1}{2}J_K^\perp\xi$. ϵ_t and ϵ_j are the scaling dimensions of the t and J_K^\perp couplings induced by the on-site interactions V and J_K^\parallel (where J_K^\parallel is the longitudinal component of the J_K -coupling). $\epsilon_j < 1$ if J_K is antiferromagnetic [4]. ϵ_t can be tuned through 1 depending on the charge-screening interactions [4,6]. The parameter [13] $M_t = F^2 K_c \xi_0^{2-\alpha_c} / 2(\alpha_c - 1)$ is generated only when the bare F value is non-zero. The scalings of ϵ_t , ϵ_j , and E_d are not explicitly affected by the M_t term. The mixed-valence regime is achieved by tuning the renormalized d -level E_d to be close to zero.

Since $\epsilon_j < 1$, we can infer from Eq. (6) that y_j is a relevant coupling. As for the usual Kondo problem, this implies the formation of quasiparticle-like spin excitation spectrum at

low energies. The resulting dynamical spin susceptibility has the usual Fermi liquid $(\frac{1}{\tau})^2$ form. What determines the nature of the charge excitations in our RG approach is the behavior of the y_t coupling. If the flow of y_t is towards infinity, the charge excitations are also quasiparticle-like and a Fermi liquid state emerges at low energies. If the flow of y_t is instead towards vanishing or an intermediate coupling, the charge excitations are distinctive from the quasiparticle-like behavior; spin-charge separation then takes place in the mixed-valence regime. In our case, y_j goes to strong coupling logarithmically. This is a sufficiently slow flow towards strong coupling that, when the initial value of $\epsilon_t + M_t$ is sufficiently larger than 1, y_t does not flow towards strong coupling in the regime where the above scaling equations are valid. This leads to the possibility for a fixed point with a weak-coupling y_t despite a strong coupling y_j .

That such a fixed point is indeed a stable one can be seen by a strong coupling analysis. We construct a Toulouse-point-like Hamiltonian by first introducing an abelian bosonization of the fermionic bath [14] and then carrying out a canonical transformation to eliminate the longitudinal couplings to these abelian bosons. In the parameter regime for the intermediate phase and in the absence of the F -coupling this procedure was carried out in Ref. [5]. In the presence of the F -coupling, the effective Hamiltonian assumes the following form,

$$\begin{aligned}
H_{eff} &= U^\dagger H_{imp}^{eff} U = H_{kin} + H_1 + H_2 \\
H_1 &= \frac{t}{\sqrt{2\pi a}} \sum_{\sigma} (X_{\sigma 0} e^{-i\Phi_c \sqrt{2}} + H.c.) - \frac{J_K^\perp}{4\pi a} (X_{\uparrow\downarrow} + H.c.) \\
H_2 &= \frac{\kappa_s}{2\pi\rho_o} (X_{\uparrow\uparrow} - X_{\downarrow\downarrow}) \left(\frac{1}{2\pi}\right) \left(\frac{d\Phi_s}{dx}\right)_{x=0} + F \sum_k \left(\sum_{\sigma} X_{\sigma\sigma} - X_{00}\right) (\rho_k + \rho_{-k}^\dagger)
\end{aligned} \tag{7}$$

Here $\Phi_{c,s} = (\Phi_\uparrow \pm \Phi_\downarrow)/\sqrt{2}$, where Φ_\uparrow and Φ_\downarrow are the Tomonaga bosons associated with the conduction electron bath, and a denotes an ultraviolet cutoff introduced in the bosonization formalism [14]. We have taken $U = e^{-i(\sqrt{2}/2)\Phi_s \sum_{\sigma} \sigma X_{\sigma\sigma}} e^{i(\sqrt{2}/4)\Phi_c (\sum_{\sigma} X_{\sigma\sigma} - X_{00})}$. The Hubbard operators are $X_{00} = |0\rangle_d \langle 0|$, $X_{\sigma 0} = |\sigma\rangle_d \langle 0| F_\sigma$, and $X_{\sigma\sigma'} = |\sigma\rangle_d \langle \sigma'| F_{\sigma'}^\dagger F_\sigma$, where F_σ^\dagger and F_σ are the ‘‘Klein operators’’ [15–17,5]. Finally, $\kappa_s = \sqrt{2}[\tan^{-1}(\pi\rho_0(V + J_K^\parallel/4)) - \tan^{-1}(\pi\rho_0(V - J_K^\parallel/4)) - \pi]$ is an infinitesimal parameter. The J_K^\perp -term is strongly relevant. Its effect is to make the bonding combination of the doubly degenerate $|\uparrow\rangle_d$ and

$|\downarrow\rangle_d$ states lower in energy than the anti-bonding combination. The t -coupling leads to fluctuations between $|0\rangle_d$ and the bonding combination of the $|\uparrow\rangle_d$ and $|\downarrow\rangle_d$ states. It renormalizes towards zero since its scaling dimension, $1 + \frac{1}{2}F^2 K_c \xi_0^{2-\alpha_c}$, is larger than one for any non-zero F . The fixed point with vanishing t and strong coupling J_K is indeed stable. Since the t -term is the only coupling that mixes the impurity spin and charge degrees of freedom, this establishes the dynamical separation of the charge and spin excitations.

The strong coupling analysis also allows us to address the effect of the g -coupling to the vector bosons in the effective impurity Hamiltonian Eq. (3). The effect of this coupling is to add a term

$$\Delta H_1 = g \sum_k [e^{i\sqrt{2}\Phi_s} X_{\uparrow\downarrow} (\phi_k^- + \phi_{-k}^{-,\dagger}) + H.c.] \quad (8)$$

where $\phi_k^- = (\phi_k^x - i\phi_k^y)/\sqrt{2}$, to H_1 defined in Eq. (7), and another term

$$\Delta H_2 = g \sum_k (X_{\uparrow\uparrow} - X_{\downarrow\downarrow}) (\phi_k^z + \phi_{-k}^{z\dagger}) \quad (9)$$

to H_2 . In this form, the coupling in ΔH_1 is less relevant (in the RG sense) than the J_K^\perp coupling. On the other hand, ΔH_2 leads to an additional screening of the impurity spin by the bosonic ϕ^z bath.

Consider $\alpha_s = 2$ first. The scaling dimension of J_K^\perp is $(\kappa_s/\pi)^2/2 + (g^2 K_s)/2$. Given that κ_s is a vanishingly small parameter, the J_K^\perp -term continues to have the strong coupling behavior for a finite range of g :

$$g^2 < g_c^2 = \frac{2}{K_s} \quad (10)$$

The t -term remains an irrelevant coupling. As a result, the spin-charge separation phenomenon remains. Physically, while the RKKY interaction introduces mutual screening between localized spins and hence inhibits the ability of the conduction electron spins to quench the local moments, over this range it is not strong enough to prevent the formation of the Kondo singlets.

While our conclusion that spin-charge separation persists for $\alpha_s = 2$ and small g is firm, if we extend our strong coupling analysis to large g and assume that ΔH_1 remains less

relevant compared to the J_K^\perp -coupling even for large g we will predict a phase transition for $\alpha_s = 2$: J_K^\perp becomes irrelevant for $g > g_c$. This phase transition is entirely the result of a competition between the g and J_K coupling. The situation is very different when *both* g and J_K are small. Here we can derive the following RG equations (involving g and J_K only) using the standard multiplicative RG procedure [19],

$$\begin{aligned} dJ_K/d\ln\xi &= J_K(\rho_0 J_K - \frac{1}{2}\rho_0^2 J_K^2 - K_s g^2) \\ dg/d\ln\xi &= -g(\frac{1}{2}\rho_0^2 J_K^2 + K_s g^2) \end{aligned} \quad (11)$$

Irrespective of the ratio g/J_K , Eqs. (11) imply that J_K always renormalizes towards strong coupling. We conclude that, if a phase transition indeed occurs in the spin-isotropic case for $\alpha_s = 2$ it can only happen at a *finite* g .

For $\alpha_s < \sim 2$, we can infer [18] from Eqs. (7,8,9) a phase transition at $g_{c,a}^2 = \frac{2}{K_s} \left(\frac{J_K^\perp \rho_0}{\sqrt{2-\alpha_s}} \right)^{(2-\alpha_s)}$. J_K^\perp flows towards strong coupling for $g < g_{c,a}$ and towards weak coupling for $g > g_{c,a}$, both in a power law fashion (in terms of the running scale ξ). The flows are controlled by an unstable fixed point at $(\sqrt{K_s/2}g^*, \rho_0 J_K^{\perp,*}) = (1, \sqrt{2-\alpha_s})$. This phase transition has its weak coupling analog. Here, the multiplicative RG equations (11) have to be modified by adding a term $(2-\alpha_s)g$ to the scaling of g . These RG equations then imply a phase transition at $g_{c,b} = \frac{2-\alpha_s}{\sqrt{K_s}} e^{-\frac{2-\alpha_s}{\rho_0 J_K^*}}$: J_K flows towards strong coupling for $g < g_{c,b}$ and towards zero for $g > g_{c,b}$. Again, the flows are algebraic in terms of ξ , and are controlled by an unstable fixed point located at $(\sqrt{K_s}g^*, \rho_0 J_K^*) = (2-\alpha_s, 2-\alpha_s)$. Through the interference effect described in Eq. (6), an algebraic flow of J_K towards strong coupling will likely make the t -coupling flow towards strong coupling. The latter would destroy the spin-charge separation phenomenon as described in this paper.

One crucial feature is that, in all of the above regimes $G_{cc}(\tau) \equiv -\langle T_\tau c_{0\sigma}(\tau) c_{0\sigma}^\dagger(0) \rangle_{H_{eff}^{imp}}$ has a long time $\frac{1}{\tau}$ behavior. This can be seen from H_{eff} , Eqs. (7,8,9), using the bosonization representation [14] $c_{0\sigma} = F_\sigma \frac{1}{\sqrt{2\pi a}} e^{-i(\Phi_c + \sigma\Phi_s)/\sqrt{2}}$.

We are now in position to determine the self-consistent solutions of the lattice Hamiltonian Eq. (1). We consider the local interactions such that the solution is the spin-

charge separated intermediate phase when the intersite interactions $v_0 = J_0 = 0$. Here, the dynamical spin susceptibility $\chi_s \sim \frac{1}{\tau^2}$ and the connected dynamical charge susceptibility $\chi_c \sim \frac{1}{\tau^\alpha}$ where α is smaller than 2. We ask whether such a solution remains a self-consistent one when J_0 and v_0 become finite. Inserting $\chi_s = \frac{1}{(E_s\tau)^2}$ and $\chi_c = \frac{1}{(E_c\tau)^\alpha}$ into the self-consistency equation (4), the corresponding impurity problem, Eq. (3), has $\alpha_s = 2$, $\alpha_c = \alpha$, $g = J_0(E_s^0/E_s)$, and $F = v_0(E_c^0/E_c)^{\alpha/2}$ (where $E_s^0 = [\lim_{\omega \rightarrow 0} \sum_q \delta(\omega - \omega_q)/\omega]^{-1/2}$ and $E_c^0 = [\lim_{\omega \rightarrow 0} \sum_q \delta(\omega - W_q)/\omega^{\alpha-1}]^{-1/\alpha}$). Also from the self-consistency, $G_{cc}(\tau) \sim \frac{1}{\tau}$ at long times implies that $\rho_0 = \sum_k \delta(E_F - E_k)$ is finite (as already noted earlier). Our analysis above then shows that, for J_0 smaller than a threshold value $(E_s/E_s^0)g_c$, the resulting dynamical spin susceptibility is also $\chi_s \sim \frac{1}{\tau^2}$. Equally important, the charge susceptibility has the form [18] $\chi_c \sim \frac{1}{\tau^\alpha}$, with the same exponent as that of the input χ_c . We conclude that, for arbitrary values of v_0 and a finite range of J_0 , the solution to the lattice model has spin-charge separation, with a local spin susceptibility of the Fermi liquid $1/\tau^2$ form and a connected local charge susceptibility that depends on $1/\tau$ through an interaction-dependent exponent. The separated spin and charge excitations is most clearly seen in the strong coupling Hamiltonian H_{eff} given in Eqs. (7, 8,9). From H_{eff} we can also infer that the single particle Green's function, $G_{dd}(\tau) \equiv - \langle T_\tau d_{0\sigma}(\tau) d_{0\sigma}^\dagger(0) \rangle_{H_{imp}^{eff}} = - \langle T_\tau d_{\sigma,eff}(\tau) d_{\sigma,eff}^\dagger(0) \rangle_{H_{eff}}$, where $d_{\sigma,eff}^\dagger = (-1/\sqrt{2})X_{\sigma_0}F_{\bar{\sigma}}e^{-i\sigma\Phi_c/\sqrt{2}}e^{i\sigma\Phi_s/\sqrt{2}}$, has a power-law dependence on $\frac{1}{\tau}$ with an interaction-dependent exponent.

The main remaining issue is the self-consistent solution when J_0 becomes sufficiently large. What is needed is a method capable of determining from the self-consistent equations the evolution of E_s towards zero. It is reasonable to assume [20] that as E_s becomes zero the local spin susceptibility has the form $\frac{1}{\tau^{\alpha_s}}$ with $\alpha_s < 2$. As discussed earlier, in this case the spin-charge separation physics will not be likely to hold. Within the spin sector alone the situation bears formal similarities with the nontrivial solutions to certain infinite-range quantum spin glass problems [21–23].

Our results provide the first step towards establishing spin-charge separation in the two-band extended Hubbard model in finite dimensions. The logical next step is to study the

effects of intersite couplings which are not two-particle in nature. However, in many cases the dominant effective intersite interactions are the two-particle RKKY-like interactions we have considered. This makes it plausible that the spin-charge separation physics we have discussed is already relevant to real materials.

The weak coupling scaling equations similar to our Eqs. (11) have been independently derived by Anirvan Sengupta, who has in addition determined the correlation functions at the unstable fixed point for $\alpha_s < \sim 2$. We would like to thank him for informing us of his unpublished results while the present paper was being written. We would also like to thank A. Georges, S. Sachdev, and A. Sengupta for useful discussions. Q. S. acknowledges the support of an A. P. Sloan Fellowship.

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$$-\sum_{i<j}(M(\alpha_i, \alpha_j)+M(\alpha_{i+1}, \alpha_{j+1})-M(\alpha_i, \alpha_{j+1})-M(\alpha_{i+1}, \alpha_j))((\frac{(\tau_j-\tau_i)}{\xi_0})^{2-\alpha_c}-1)/(2-\alpha_c),$$

where $M(\sigma, 0) = M(0, \sigma) = -M_t = -F^2 K_c \xi_0^{2-\alpha_c} / 2(\alpha_c - 1)$ and $M(\sigma, \sigma') = 0$. When g -coupling is also finite, the contribution to the partition function associated with a particular history of the impurity configurations, while still calculable, doesn't appear to have the form of a Coulomb-gas action.

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